Labeling Curves with Curved Labels

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Labeling places and features is an important topic in cartography [1]. As well as improving readability, labels can convey place or feature *type* and *hierarchy* through their position, orientation, size, colour, typeface and style. These characteristics also have aesthetic impacts on the map which cartographers and designers use to affect how people engage and interpret maps.

Curved labels are often used on maps. Places (point features) around a coastline may benefit from curved labels that do not interfere with surrounding linework (Fig. 1a). Labels that follow linear features (Fig. 1b) help improve their association (compare with Fig. 1c). Curved labels can also help indicate the shape of aerial features (Fig. 1e) and can fit into the space more effectively (Fig. 1d). Curvature as a label *style* may also help distinguish different *types* of features (Fig. 1e). Although there are benefits to curved labels, curvature should be kept low and symmetrical where possible to improve legiblity [2].

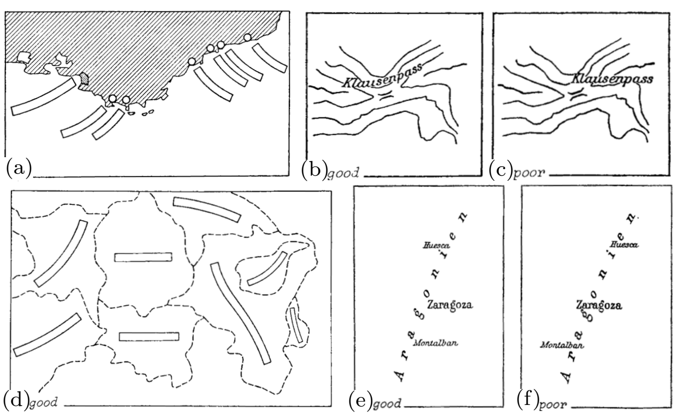

Fig. 1: Imhof's [1] examples of appropriately curved labels for (a) point features, (b–c) a feature that shares point and linear characteristics, (d) curved areal features and (e–f) point features on an areal feature.

We consider the specific case of labeling point features on curves, inspired by metro maps that use Bézier curves [3]. As well as curved labels improve packing so there's more potential to fit more labels on such maps (Fig. 2a–2c), we also wish the labels to more closely match the curved style of the map for aesthetic reasons.

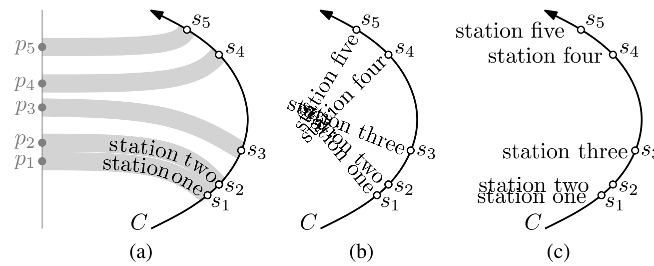

Fig. 2: (a)–(c) Different types of labels for the same metro line C. (a) Curved labels connecting stops of curve C with ports to the left of C. (b) Using straight labels perpendicular to C yields unavoidable clashes. (b) Straight labels can become cluttered when aligned horizontally.

I. Model for Labeling Curves

In order to simplify the problem of labeling metro maps we start with only one metro line and assume that its shape is already given by a directed, smooth, non-self-intersecting curve C in the plane, for example described by a Bézier curve. Further, the stops of the metro line are given by an ordered set S of points on C going from the beginning to the end of C; see Fig. 2a.

For each stop we are further given a name that should be placed closely to it. In contrast to previous work, e.g., [4], [5], [6], [7], we do not follow traditional map labeling abstracting from the given text by bounding boxes, but we use fat curves prescribing the shape of the labels; see Fig. 2a.

Hence, for each stop $s \in S$ we want to find a fat curve ℓ of fixed width and given length such that 1) ℓ begins at s, 2) ℓ does not intersect C, and 3) ℓ does not intersect any other fat curve ℓ' for any $s' \in S \setminus \{s\}$. We call these curves *curved labels*. To obtain a model that on the one hand is reasonable and on the other hand promises combinatorial algorithms solving optimization problems for labeling curves, we further assume that we are given a set P of points that do not lie on C; we call those points *ports*. For each stop $s \in S$ and each port $p \in P$ we are then given a curved label $\ell(s, p)$ that begins at s, ends at p and whose shape is already fixed. Thus, for a single stop $s \in S$ the set P induces a set of

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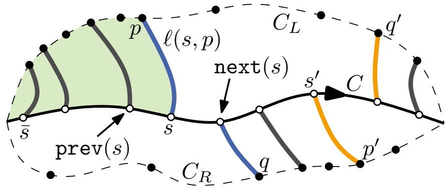

Fig. 3: The metro line C enclosed by C_L and C_R. The green area shows an instance of the one-sided case. The label $\ell(s,p)$ separates that instance from the remaining one-sided instance above C. For the two sided case (s,p,q) is a switchover.

candidate labels that can be placed for labeling the stop s. Note that the algorithms that are presented in the following do not rely on the actual shape of the labels. However, to avoid labels describing wavy lines one may model a curved label $\ell(s,p)$ as a cubic Bézier curve.

We further make the assumption that the metro line C is enclosed in a simple, closed region R such that only the end points of C lie on the border of R, see Fig. 3. Consequently, C divides the border of R into two non-self-intersecting curves C_L and C_R. We require that the ports in P lie on C_L and C_R. For a stop $s \in S$ and a port $p \in P$ a label $\ell(s,p)$ is *valid* if its name fits into $\ell(s,p)$ and if it neither intersects C, C_L nor C_R.

Fixing the start and end of C, we denote for a stop $s \in S$ its successive stop by $\texttt{next}(s)$ and its antecendent stop by $\texttt{prev}(s)$. Consequently, the stops in S form a linked list. We denote the first stop of that list by $\bar{s}$ and the sub-list from the stop s to the stop s' by $S(s,s')$. If s lies before s' on C we also write $s < s'$.

A *valid* labeling $\mathcal{L}$ of an instance (S,P) is then a set of valid labels such that the curved labels in $\mathcal{L}$ do not intersect each other and $\mathcal{L}$ describes an injective function from S to P, i.e., each stop is connected with one port and each port is connected with at most one stop. For two valid labels $\ell(s,p)$ and $\ell(\texttt{next}(s),q)$ of two consecutive stops s and $\texttt{next}(s)$, we call (s,p,q) a *switchover* if p and q do not lie on the same border $B \in \{C_L, C_R\}$. We are then interested in a valid labeling $\mathcal{L}$ of an instance (S,P) such that $\mathcal{L}$ has the minimum number of switchovers under all valid labelings of (S,P).

II. Labels on One Side of the Curve

We first assume that all ports lie either on C_L or C_R. Later on we use the one-sided case to solve the two-sided case. We now describe a dynamic programming approach based on a two-dimensional table T containing binary values. The first dimension corresponds with the given stops and the second dimension with the given ports. The idea is that $T[s,p] = 1$ for $s \in S$ and $p \in P$ if the instance $(S(\bar{s},s),P)$ has a valid labeling $\mathcal{L}$ such that $\ell(s,p)$ belongs to that labeling, and otherwise, $T[s,p] = 0$. Note that $\ell(s,p)$ splits the instance (S,P) into two sub-instances, which in particular means that the stops $S(\bar{s},s)$ can only be connected to ports that lie in the same sub-instance as $S(\bar{s},s)$; see Fig. 3.

For $\bar{s}$ and each port $p \in P$ we set $T[\bar{s},p] = 1$ if $\ell(\bar{s},p)$ is valid and otherwise $T[\bar{s},p] = 0$. For each station $s \in S \setminus \{\bar{s}\}$ and each port $p \in P$ let $W(s,p)$ be the set of ports $q \in P \setminus \{p\}$ for which $\ell(\texttt{prev}(s),q)$ is valid, and $\ell(s,p) \cap \ell(\texttt{prev}(s),q) = \emptyset$. Then, we define

$$T[s,p] = \max\{0, T[\texttt{prev}(s),q] \mid q \in W(s,p)\}$$

The instance (S,P) has a valid labeling $\mathcal{L}$ if and only if there is a port p such that $T[s,p] = 1$ for the last station s on C. Using backtracking $\mathcal{L}$ can be constructed from T. Setting $n = |S|$ and $m = |P|$ we obtain the following theorem.

Theorem 1: In the one-sided case, the existence of a valid labeling can be checked in $O(nm^2)$ time and $O(n+m)$ space.

III. Labels on Both Sides of the Curve

If the ports lie on both C_L and C_R, we can solve the problem utilizing the algorithm for the one-sided case. For each possible switchover (s',p',q') we systematically seek the switchover (s,p,q) with $s < s'$ that lies directly before (s',p',q'), i.e., no other switchover lies in between both; see Fig. 3. Roughly spoken, (s,p,q) and (s',p',q') induce a two-sided instance that lies to the left of (s,p,q) and a one-sided instance that lies in between (s,p,q) and (s',p',q'). Based on this observation, we can formulate a dynamic program solving the two-sided case.

Theorem 2: An optimal valid labeling can be computed in $O(n^2m^4)$ time and $O(nm^2)$ space.

IV. Conclusion

We conjecture that instances (S,P) are much more complaisant in practice than assumed for the worst-case analysis. However, experiments are still future work. Further work comprises the questions how ports can be placed, how the dynamic programs can be tuned and how more than one metro line can be handled. Considering multiple metro lines raises the question whether the problem becomes NP-hard.

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