Assignment 13

Post Date: 29 Jan 2018   Due Date: 05 Feb 2018, 11:30
You are permitted and encouraged to work in groups of two.

Problem 1: Interior of a Simple Polygon

Show how to determine efficiently whether a given point \( q \in \mathbb{R}^2 \) is in the interior of a simple polygon \( P \). For simplicity you may assume that \( q \) does not have the same x-coordinate as any vertex of \( P \).

Problem 2: Pair of Furthest Points

Let \( P \) be a set of points in the plane and let \( p_1, p_2 \in P \) be two points with the greatest distance. Show that \( p_1 \) and \( p_2 \) are on the boundary of the convex hull of \( P \).

Problem 3: Binary Search Tree

You have stored a linear order of a set of \( n \) items in a binary search tree \( T \). For each item you have direct access to the respective node in \( T \). For each node of \( T \) you have the pointers \( \text{parent}[v], \text{left}[v], \text{right}[v] \). You do not have any other information about the linear order. Develop an algorithm that outputs the successor of a given item if one exists in time proportional to the height of \( T \).

Problem 4: Shamos & Hoey

(a) Consider the following extension of the algorithm of Shamos & Hoey:
Whenever the algorithm finds an intersection it does not stop but saves the intersection to a list and continues.
Disprove the following statements:
   i. The list contains all intersections.
   ii. The intersections in the list are ordered by their \( x \)-values.

(b) Extend the algorithm of Shamos & Hoey such that it outputs all intersections according to their appearance on the \( x \)-axis. Assume that no two endpoints are equal and that at most two line segments intersect in one point.
Provide your algorithm in pseudo-code and analyze its run time.