Assignment 4

Post Date: 13 Nov 2017    Due Date: 20 Nov 2017, 11:30
You are permitted and encouraged to work in groups of two.

Problem 1: Union-Find with Path Compression 5 Points

Consider FIND with the following alternative path compression: After traversing the path from a vertex to its root, we update the parent pointer of each vertex along the path to point to its grandparent. Consider, e.g., subpath

\[ i \rightarrow j \rightarrow k \rightarrow l \rightarrow \cdots \]

Performing FIND(i) with alternative path compression results in k being predecessor of i and l being predecessor of j. Direct successors of the root keep the root as predecessor.

Go through the proof of the Theorem of Hopcroft & Ullman and find the inferences that require FIND to be implemented with path compression. Is the proof still correct if the alternative path compression is used?

Problem 2: Independent Vertex Sets 5 Points

Let \( G = (V, E) \) be a graph. Let \( I = \{ V' \subseteq V; \{u, v\} \notin E \text{ for } u, v \in V' \} \).

(a) Show that \((V, I)\) is an independence system.

(b) Is \((V, I)\) also a matroid? Justify your answer.
Problem 3: Minimum Spanning Tree of France 4 Points

Create manually the minimum spanning tree of the cities of France indicated in the map (taken from https://commons.wikimedia.org/wiki/File:France-CIA_WFB_Map.png). Explain your algorithm and why it is an application of red and blue rules.

Problem 4: Unique Minimum Spanning Trees 6 Points

(a) Prove: Let $G$ be a connected graph with real-valued edge weights. If for each cut the crossing edge with the lightest weight is unique, then $G$ has a unique minimum spanning tree.

(b) Does the inverse of the implication in (a) hold?

(c) Prove or disprove: If in a connected graph $G$ with real-valued edge weights all edges have pairwise distinct edge weights, then $G$ has a unique minimum spanning tree.

(d) Does the inverse of the implication in (c) hold?