Assignment 2

Post Date: 30 Oct 2017    Due Date: 06 Nov 2017, 11:30
You are permitted and encouraged to work in groups of two.

Problem 1: Fraud Detection 6 Points

Assume you have \( n \) bank cards. Each bank card is associated with a bank account. Different bank cards might be associated with the same bank account. The associated bank is stored encrypted on your bank card. However, you have a device that can check whether two bank cards are associated with the same bank account. The task is to find out whether there is a set of at least \( n/2 \) bank cards associated with the same bank account. Show how to decide this question by comparing at most \( O(n \log n) \) pairs of bank cards?

Problem 2: Threshold 4 Points

Give the greatest \( a \in \mathbb{N} \) such that Algorithm \( A \) with running time

\[
T_A(n) = a \cdot T_A \left( \frac{n}{9} \right) + n^2
\]

is asymptotically faster than Algorithm \( B \) with running time

\[
T_B(n) = 11 \cdot T_B \left( \frac{n}{3} \right) + n^2.
\]

Problem 3: Regularity Condition 5 Points

Let \( a \geq 1, \ b > 1, \) and \( f : \mathbb{N} \rightarrow \mathbb{R}_{>0}. \) Show that \( f(n) \in \Omega(n^{\log_b a + \epsilon}) \) for some \( \epsilon > 0 \) if there is a \( c < 1 \) and an \( n_0 \in \mathbb{N} \) such that \( a \cdot f([n/b]) \leq c \cdot f(n) \) for all \( n \geq n_0. \)

Problem 4: Matrix Multiplication 5 Points

The multiplication of two matrices \( A, B \in \mathbb{R}^{n \times n} \) where \( n = 2^m \) for some \( m \in \mathbb{N}, \) as suggested by Strassen, yields the following recurrence for the number of arithmetic operations:

\[
T(n) = \begin{cases} 
    n^3 + n^2(n - 1) & \text{if } n \leq 2^{k_0} \text{ for some const. } k_0 \geq 0 \\
    7 \cdot T \left( \frac{n}{2} \right) + 18 \cdot \left( \frac{n}{2} \right)^2 & \text{otherwise.}
\end{cases}
\]

Determine \( k_0 \in \mathbb{N} \) and a possibly small constant \( c, \) such that \( T(n) \leq c \cdot n^{\log_2 7} \) for all \( n \geq 2^{k_0}. \)

*Hint:* Trace the proof of the first case of the Master theorem.