Assignment 13

Post Date: 30 Jan 2016   Due Date: 06 Feb 2016, 1 pm
You are permitted and encouraged to work in groups of two.

Problem 1: Points Inside Simple Polygons  
You are given \( n \) points \( p_1, \ldots, p_n \in \mathbb{R}^2 \) in the plane and the line segments \( p_ip_{i+1} \) for \( i = 1, \ldots, n - 1 \) and \( p_np_1 \) form a simple polygon \( P \). A polygon is simple if none of its delineating line segments intersect; in particular, no more than two line segments meet at any point.

Derive a linear-time algorithm that tests if a point \( q \in \mathbb{R}^2 \) is inside the polygon \( P \).

Problem 2: Binary Search Tree

You are given a binary search tree \( BST \) that stores a strict total order of natural numbers of size \( n \) with \( V := \{v_1, \ldots, v_n \in \mathbb{N} \mid \forall i \in [1, n - 1] : v_i < v_{i+1} \} \).

Provide an algorithm in pseudocode that returns \( v_{i+1} \) for an arbitrary tree node \( T_i \) solely based on the structure of the \( BST \) (i.e. only using =-operators, especially no <, >, \( \leq \) or \( \geq \)). Assume self-explanatory pointers \( T.parent, T.left, T.right \) to be given.

Problem 3: Shamos & Hoey

(a) Consider the following extension of the algorithm of Shamos & Hoey:
Whenever the algorithm finds an intersection in line 7 or 9 it does not stop but saves the intersection to a list and continues.
Disprove the following statements:
i. The list contains all intersections.
ii. The intersections in the list are ordered by their \( x \)-values.

(b) Expand the algorithm of Shamos & Hoey such that it outputs all intersections according to their appearance on the \( x \)-axis. Assume that no two endpoints are equal and that at most two line segments intersect in one point.
Provide your algorithm in pseudocode and analyze its run time.
Hint: Define a new event-point-type that represents intersections of line segments.