Assignment 2

Post Date: 31 Oct 2016       Due Date: 07 Nov 2016, 1 pm
You are permitted and encouraged to work in groups of two.

Problem 1: Threshold 5 Points

Give the greatest $a \in \mathbb{N}$ such that Algorithm $A$ with running time

$$T_A(n) = a \cdot T_A\left(\frac{n}{9}\right) + n^2$$

is asymptotically faster than Algorithm $B$ with running time

$$T_B(n) = 11 \cdot T_B\left(\frac{n}{3}\right) + n^2.$$

Problem 2: Matrix Multiplication 8 Points

The multiplication of two matrices $A, B \in \mathbb{R}$ where $n = 2^m$ for some $m \in \mathbb{N}$, as suggested by Strassen, yields the following recurrence:

$$T(n) = \begin{cases} n^3 + n^2(n - 1) & \text{if } n \leq 2^{k_0} \text{ for some const. } k_0 \geq 0 \\ 7 \cdot T\left(\frac{n}{2}\right) + 18 \cdot \left(\frac{n}{2}\right)^2 & \text{otherwise.} \end{cases}$$

(a) Depending on $k_0$ determine a constant $c$, such that $T(n) \leq c \cdot n^{\log_2 7}$ for all $n > 2^{k_0}$.

Hint: Trace the proof of the first case of the Master theorem.

(b) Show how to choose $k_0$ such that the constant $c$ in

$$T(n) \leq c \cdot n^{\log_2 7}$$

is minimal. Give the constant.

(c) How can you extend the algorithm for arbitrary $n$ that are not a power of two.

Problem 3: Selection 7 Points

Does the algorithm SELECT still work in linear time if the input elements are divided into groups of 3 and 7, respectively? Prove your statements.