Assignment 1

Post Date: 24 Oct 2016   Due Date: 31 Oct 2016, 1 pm
You are permitted and encouraged to work in groups of two.

Problem 1: Growth of Functions 8 Points

Rank the functions

\[ n \log n, \left( \frac{2n}{4} \right), 2^n, 4n^4, n^n, 16^{\log_2 n}, \log(n!), n! \]

by increasing order of growth, i.e., find an order \( f_1, \ldots, f_7 \) with \( f_1 \in O(f_2), \ldots, f_6 \in O(f_7) \), and prove the correctness of your ranking.

Problem 2: Recurrence Equations 5 Points

Give a \( \Theta \)-bound for the following recurrences:

(a) \[ T_1(n) = 2 \cdot T_1 \left( \frac{n}{2} \right) + \sqrt{n}, \quad T_1(1) = 1 \]
(b) \[ T_2(n) = T_2(n-1) + 2(n-1), \quad T_2(1) = 1 \]

Problem 3: Divide and Conquer 7 Points

Let \( n \) points in the plane be given. You may assume for simplicity that no two points have the same \( x \)-coordinate. Develop a divide-and-conquer algorithm that finds a pair of points with the smallest Euclidean distance between them. Analyze the run time of your algorithm. Can you achieve a run time in \( O(n \log n) \)?

Hint: It may be helpful to think first about the 1D version of the problem. How can the merge step be realized in linear time in 2D?