Assignment 12

Post Date: 25 Jan 2016  Due Date: 01 Feb 2016, 1 pm
You are permitted and encouraged to work in groups of two.

Problem 1: Intersections of Straight Lines  
Consider a given set of \( n \) distinct straight lines in the plane.

(a) How many intersections can there be at most? Prove your statement.

(b) Find a procedure to construct a set of \( n \) straight lines with maximum number of intersections.

(c) How many intersections can there be at most if there are only horizontal or vertical lines? Prove your statement.

Problem 2: Binary Search Tree
You are given a binary search tree \( BST \) that stores a strict total order of natural numbers of size \( n \) with \( V := \{v_1, ..., v_n \in \mathbb{N} \mid \forall i \in [1, n-1] : v_i < v_{i+1}\} \).

Provide an algorithm in pseudocode that returns \( v_{i+1} \) for an arbitrary tree node \( T_i \) solely based on the structure of the \( BST \) (i.e. only using \( = \)-operators, especially no \( <, >, \leq \) or \( \geq \)). Assume self-explanatory pointers \( T.parent, T.left, T.right \) to be given.

Problem 3: Shamos & Hoey

(a) Consider the following extension of the algorithm of Shamos & Hoey:
Whenever the algorithm finds an intersection in line 7 or 9 it does not stop but saves the intersection to a list and continues.
Disprove the following statements:
   i. The list contains all intersections.
   ii. The intersections in the list are ordered by their \( x \)-values.

(b) Expand the algorithm of Shamos & Hoey such that it outputs all intersections according to their appearance on the \( x \)-axis. Assume that no two endpoints are equal and that at most two line segments intersect in one point.
Provide your algorithm in pseudocode and analyze its run time.
Hint: Define a new event-point-type that represents intersections of line segments.