Assignment 5

Post Date: 16 Nov 2015  Due Date: 23 Nov 2015, 1 pm
You are permitted and encouraged to work in groups of two.

Problem 1: Minimum Spanning Tree 7 Points

Find an MST for the graph on the right using

(a) the coloring method of Tarjan,
(b) the algorithm of Kruskal, and
(c) the algorithm of Prim.

Indicate in each step the edge that has to be colored together with the corresponding color. When you apply the algorithm of Prim, use a Fibonacci heap and give for each step the necessary heap operations and show how the heap looks like.

Problem 2: Height of a Fibonacci Heap 4 Points

Prove or disprove that the height of a tree of a Fibonacci Heap with $n$ vertices is in $O(\log n)$.

Problem 3: Independent Vertex Sets 4 Points

Let $G = (V, E)$ be a graph. Let $\mathcal{I} = \{V' \subseteq V; \{u, v\} \notin E \text{ for } u, v \in V'\}$.

(a) Show that $(V, \mathcal{I})$ is an independence system.
(b) Is $(V, \mathcal{I})$ also a matroid? Justify your answer.

Problem 4: Weight Sequence 5 Points

Let $(X, \mathcal{I})$ be a matroid and let $\omega : X \rightarrow \mathbb{R}$ be a weight function. The weight sequence of a basis $B = \{x_1, \ldots, x_d\}$ of $(X, \mathcal{I})$ is the sequence $\langle \omega(x_1), \ldots, \omega(x_k) \rangle$ of weights of the elements of $B$ ordered such that $\omega(x_1) \leq \cdots \leq \omega(x_k)$.

Show that any two minimum weight bases of a matroid have the same weight sequence.