Assignment 8

Post Date: 13 Dec 2013  Due Date: 20 Dec 2013, 08:00 (AM)
You are permitted and encouraged to work in groups of two.

Problem 1: Minimum Cut 8 Points

(a) Find a minimum cut for the following graph with the algorithm of Stoer and Wagner. Start with vertex a and comment each step of the algorithm.

![Graph Image]

(b) Find an example graph with negative edge weights such that the algorithm of Stoer and Wagner does not find a minimum cut.

Problem 2: Santa Claus Problem 8 Points

In order to have enough presents for all children, Santa Claus prepares presents all year round. At the beginning of December he obtains wish lists from all the children. Now Santa Claus tries to assign each child a present such that as many children as possible obtain a present they are happy with.

(a) Formulate the problem as a maximum flow problem. Show that an optimum solution of Santa Claus’ problem can indeed be constructed from a maximum flow in your flow network.

(b) For a child c let \( W(c) \) be the set of presents on its wish list. For a set \( C \) of children let \( W(C) = \bigcup_{c \in C} W(c) \). Use the max-flow min-cut theorem to show that Santa Claus can assign the presents such that all children receive a present they like if and only if there is no subset \( C \) of children with \( |W(C)| < |C| \).

[please turn over]
Problem 3: 4 Points

We are given a set of jobs \( j \in J, |J| < \infty \), and for each task \( j \) a processing time \( p_j \in \mathbb{R}_0^+ \), an earliest starting time \( r_j \in \mathbb{R}_0^+ \) and a deadline \( d_j \geq r_j + p_j \), as well as \( M \) machines. At a point in time, each machine can process only one job and each job can be processed by one machine only. However, jobs can be paused and continued later at the same or another machine.

The goal is to decide whether there is a processing order of the jobs on the machines, such that the schedule properties (above) are maintained.

Show how to reduce this problem to a max-flow problem, and create the flow network for the following example: \( M = 3, J = \{1, 2, 3, 4\} \), and 

\[
\begin{align*}
  p_1 &= 1.5, & r_1 &= 3, & d_1 &= 5, \\
p_2 &= 1.25, & r_2 &= 1, & d_2 &= 4, \\
p_3 &= 2.1, & r_3 &= 3, & d_3 &= 7, \\
p_4 &= 3.6, & r_1 &= 5, & d_4 &= 9.
\end{align*}
\]

Justify your answer.