Assignment 9

Post Date: 19 Dec 2012  Due Date: 9 Jan 2013, 14:30
You are permitted and encouraged to work in groups of two.

Throughout these assignments, we denote by boundary function the function
\[ B(j) = \begin{cases} -1, & j = 0 \\ |\partial(P[1, \ldots, j])| = \text{sup}_P(P[2, \ldots, j]), & j = 1, \ldots, m \end{cases} \]
assigning to the length of a prefix of a pattern \( P[1, \ldots, m] \) the length of its proper boundary.

Problem 1: Knuth-Morris-Pratt and Boyer-Moore 8 Points

(a) Compute the boundary function for the pattern
\[ P = \text{ababbaababababbab} \]
(b) Compute the bad character function and the good suffix function for the alphabet
\( \Sigma = \{0, 1, 2, 3\} \) and the pattern
\[ P = 0101101201. \]

Problem 2: Transition Function 8 Points

Let \( \delta \) be the transition function of a pattern \( P[1, \ldots, m] \).

(a) Show that \( \delta(q, a) = \delta(B[q], a) \) for any \( a \in \Sigma \) and \( 0 < q \leq m \) with \( q = m \) or \( P[q + 1] \neq a \).
(b) Give an \( \mathcal{O}(m|\Sigma|) \)-time algorithm for computing the transition function \( \delta \) corresponding to a given pattern \( P \) of length \( m \).

Problem 3: Repetition Factor 4 Points

Let \( P \) be a pattern of length \( m \). For a \( q = 1, \ldots, m \) let
\[ \rho(q) = \max \{r; P[1, \ldots, q] = x^r \text{ for some } x \in \Sigma^*\} \]
Prove or disprove that \( \rho(q) > 1 \) if and only if there is an \( i > 0 \) with \( B^i(q) > 0 \) and
\[ q - B^i(q) = \frac{B^i(q)}{\rho(B^i(q))}. \]

Merry Christmas and a happy New Year!