Assignment 3

Post Date: 07 Nov 2012  Due Date: 14 Nov 2012, 14:30
You are permitted and encouraged to work in groups of two.

Problem 1: Selection 6 Points

Does the algorithm Select still work in linear time if the input elements are divided into groups of 3 and 7, respectively? Prove your statements.

Problem 2: Binary Counter 7 Points

An array $A$ of length $k$ is used to implement a $k$-bit binary counter that counts upwards from zero. A binary number stored in the counter has its lowest-ordered bit in $A[0]$ and its highest-order bit in $A[k-1]$. The counter supports the operation $\text{Reset}(A)$ that sets all bits to 0 and the operation $\text{Increment}(A)$ implemented as follows:

\begin{verbatim}
Algorithm 1 Increment(A)
    i ← 0
    while $i < \text{length}[A]$ and $A[i] = 1$ do
        A[i] ← 0
        i ← i + 1
    end while
    if $i < \text{length}[A]$ then
        A[i] ← 1
    end if
\end{verbatim}

Give an upper bound for the running time of one $\text{Reset}$ and $n$ $\text{Increment}$ operations

(a) based on the worst-case running time for each individual operation

(b) based on an amortized analysis of the $n + 1$ operations.
Consider a heap realized as an array $A$ with operations INSERT and DELETEMIN. The worst-case costs of both operations are in $O(\log n)$ where $n$ is the number of elements in the heap.

(a) Give a potential function $C$ such that the amortized costs of an INSERT-operation with respect to $C$ are in $O(\log n)$ and the amortized costs of a DELETEMIN-operation with respect to $C$ are in $O(1)$. Assume that the heap is empty at the beginning.

(b) The length of $A$ can be dynamically adapted to the number of elements in the heap:

- Let the current length of $A$ be $2^k - 1$ for a $k \in \mathbb{N}$.
- If $A$ is completely filled, all elements in $A$ are moved to a new array of length $2^{k+1} - 1$, before the $2^k$th element is added.
- If $A$ contains only $2^{k-2}$ elements, all elements in $A$ are moved to a new array of length $2^{k-1} - 1$, after the $2^{k-2}$th element is deleted.

Start with an empty heap and assume that the costs to move an element from one array to another are in $O(1)$. What are the amortized costs of $m$ INSERT-and DELETEMIN-operations?