Assignments $N^o$ 2 - PART II

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Task 1: Inappropriate Sampling from ERGMs

So far, algorithms for sampling from $G(n,p)$ or the preferential attachment model decided about the inclusion/exclusion of edges one after the other.

Provide a (preferably simple) example which demonstrates that a corresponding strategy does not work out for the ERGM class in general, i.e. starting with the empty edge set $E = \emptyset$ and sequentially adding edge $e$ to $E$ with probability $\frac{P(V,E \cup \{e\})}{P(V,E) + P(V,E \cup \{e\})}$ yields incorrect outcomes — you are allowed to reuse probability calculations from the lecture.

Task 2: Appropriate Sampling from ERGMs

Let $\mathcal{G}$ the set of undirected, loopless graphs with $n = 3$ vertices and consider the exponential random graph model $(\mathcal{G}, P)$ containing only the number of two-stars statistic $g_1 = s_2$ with parameter $\theta_1 = \ln 2$.

According to the Gibbs sampling strategy defined in the lecture, specify the transition probabilities $\pi$ in a Markov chain on $\mathcal{G}$ with unique stationary distribution $P$.

Note that you don’t have to provide all $8 \times 8$ transition probabilities explicitly, since there is no need to distinguish between isomorphic graphs, and many transition probabilities are 0. Consequently, present your result within a single graph in which the nodes represent the equivalence classes of isomorphic networks in $\mathcal{G}$ and edges are labeled according to the positive transition probabilities.