Assignment 11

Post Date: 14 Jan 2009   Due Date: 21 Jan 2009, 14:30
You are permitted and encouraged to work in groups of two.

Problem 1: Shamos & Hoey 10 Points

(a) Consider an extension of the algorithm of Shamos & Hoey. If the algorithm finds an intersection in line 7 or 9, then it does not stop but saves the intersection in a list and continues the algorithm. Assume that no two endpoints are equal.

Disprove the following statements:

i. The list contains all intersections.

ii. The intersections are in the list according to their appearance on the $x$-axis.

(b) Expand the algorithm of Shamos & Hoey such that it outputs all intersections according to their appearance on the $x$-axis. Write a pseudocode for your algorithm and analyze the running time.

Assume that no two endpoints are equal and that at most two line segments intersect in one point.

Advice: Define a new event-point-type that represents crossings of line segments.

Problem 2: Simple Polygons 2 Points

Find an algorithm that tests in $O(n \log n)$ time if a sequence of points in the plane is the sequence of the vertices of a simple polygon.
Problem 3: Divide-and-Conquer Convex Hull Algorithm

Describe a recursive algorithm to compute the convex hull of a set of $n$ points $P = \{p_1, \ldots, p_n\}$ in the plane in $O(n \log n)$ time. To do so structure your algorithm roughly around the following steps.

(a) Divide the $n$ points $p_1, \ldots, p_n$ into two disjoint sets $P_1$ and $P_2$ of size $\lceil n/2 \rceil$ and $\lfloor n/2 \rfloor$, respectively.

(b) Recursively compute the convex hulls $C_1$ of $P_1$ and $C_2$ of $P_2$.

(c) Merge $C_1$ and $C_2$ into the convex hull of $P$.

Hints: The difficulty lies in the merge step; mention the time-complexity that must be achieved for this step such that an overall bound of $O(n \log n)$ follows. Use the fact that the points in $C_1$ and $C_2$ are ordered counterclockwise.

To implement merge choose a point $p^*$ in the interior of $C_1$ and distinguish two cases, namely whether $p^*$ lies in the interior of $C_2$ or not. Merge the points of $C_1$ and $C_2$ in a suitable manner.