Assignment 4

Post Date: 12 Nov 2008   Due Date: 19 Nov 2008, 14:30
You are permitted and encouraged to work in groups of two.

Problem 1: Union by Rank 6 Points

Union by rank (h-Union) is to always attach the tree with the smaller height to the root of the tree with the larger height. Let $T$ be a tree with $n$ vertices and height $h$ that is created by a sequence of MAKESET and h-Union operations.

(a) Does the inequality $h \leq \log_2 n$ hold?

(b) Exists for all $n$ a sequence of MAKESET and h-Union such that $h \geq \lfloor \log_2 n \rfloor$?

Problem 2: Union with Path Compression 3 Points

Give a pseudocode for FIND with path compression similar to the pseudocode of FIND without path compression from the lecture.

Problem 3: Sequence of Operations 8 Points

In a sequence of $n$ operations MAKESET, FIND with path compression, and weighted UNION all UNION operations are done before the first FIND operation is executed.

(a) Show that the cost for $n$ operations are in $O(n)$.

(b) What if FIND is still with path compression but UNION is unweighted?

(c) What if FIND is without path compression and UNION is weighted?

Problem 4: Potential Function 3 Points

Let $\phi$ be a potential function such that $\phi(D_i) \geq \phi(D_0)$ for all $i$. Show that there exists a potential function $\phi'$ with $\phi'(D_0) = 0$ and $\phi'(D_i) \geq 0$ for all $i \geq 1$ such that the amortized cost of $\phi'$ and $\phi$ are the same.