Assignment 3

Post Date: 05 Nov 2008 Due Date: 12 Nov 2008, 14:30
You are permitted and encouraged to work in groups of two.

Problem 1: Aggregate Analysis 4 Points

The $i$-th operation of a sequence of $n$ operations that are performed on a data structure costs $i$ if $i$ is an exact power of 2, and 1 otherwise. Determine with aggregate analysis the amortized cost per operation.

Problem 2: Binary Counter 6 Points

Use an array $A$ of length $k$ to implement a $k$-bit binary counter that counts upward from 0. A binary number stored in the counter has its lowest-ordered bit in $A[0]$ and its highest-order bit in $A[k-1]$. To add 1 to the value in the counter, use the following procedure:

Algorithm 1 INCREMENT($A$)

\[
\begin{align*}
  i &\leftarrow 0 \\
  \text{while } i < \text{length}[A] \text{ and } A[i] = 1 \text{ do} \\
  &\quad A[i] \leftarrow 0 \\
  &\quad i \leftarrow i + 1 \\
  \text{end while} \\
  \text{if } i < \text{length}[A] \text{ then} \\
  &\quad A[i] \leftarrow 1 \\
  \text{end if}
\end{align*}
\]

(a) What is the worst-case running time for $n$ INCREMENT operations on an initially zero counter?

(b) Show with one of the three techniques used in amortized analysis a linear upper bound.
Problem 3: Dynamic Tables  10 Points

(a) Table Expansion:

• When an item is inserted into a full table, allocate a new table that has twice as many slots as the old one. Determine the worst-case upper bound if $n$ operations are performed.

• The bound above is not tight. Determine the amortized cost of a Table-Insert operation using the potential method.

  Advice: Start by defining a potential function $\Phi$ that is zero immediately after an expansion but builds to the table size by the time the table is full.

(b) Table Expansion and Contraction:

• What happens if one doubles the table size when an item is inserted into a full table and halve the size when a deletion would cause a table to become less than half full? Determine a bad scenario of Table-Insert and Table-Delete operations.

• Can you think of a better strategy? You do not have to give a detailed proof for the amortized cost of your idea.