Assignment 1

**Post Date:** 13 April 2015  **Due Date:** 20 April 2015  **Tutorial:** 29 April 2015

**Problem 1: Different Embeddings**  
3 Points

Let $G$ be a planar graph with a planar embedding with $f$ faces. The *degree* of a face $f_i$ is the number $a_i$ of edges that are incident to $f_i$ (counting bridges twice). Assume that the faces $f_1, \ldots, f_f$ are ordered such that their degrees are non descending. Consider the face-degree-sequence $(a_1, \ldots, a_f)$.

Can a planar graph have two embeddings with different face-degree-sequences?

**Problem 2: Euler’s Formula**  
4 Points

Let $n$ be the number of vertices, $m$ the number of edges, $f$ the number of faces and $k$ the number of connected components of a planarly embedded simple graph $G$.

(a) Prove by induction on $m$ that $f - m + n - k = 1$.

(b) Conclude that $m \leq 2n - 4$ and $f \leq n - 2$ if $G$ is bipartite and $n \geq 3$.

**Problem 3: Vertex Coloring**  
3 Points

Give a linear time algorithm that colors the vertices of a planar graph with at most six colors such that no two adjacent vertices have the same color.