Assignment 9

Post Date: 20 June 2014   Due Date: 27 June 2014   Tutorial: 2 July 2014
You are permitted and encouraged to work in groups of two.

Problem 1: Alternating and Augmenting Paths 5 Points

Find the following in the graph below. Matched edges are drawn thicker.

(a) an alternating path of length 10
(b) an alternating circle of length 10
(c) an augmenting path of length 5
(d) an augmenting path of length 9
(d) an augmenting tree with $v_2$ as root
Problem 2: Max Weighted Matching

Given a graph $G = (V, E)$ with edge weights $w : E \rightarrow \mathbb{R}$, we want to find a matching $M$, with maximum weight $w(M) := \sum_{e \in M} w(e)$.

Consider the following transformation of an instance of max weighted matching into an instance of min perfect matching:

We construct a graph $G^* = (V^*, E^*)$ from $G$ as follows. For every node $v \in V$, we create two nodes $v'$ and $v''$ in $V^*$. For every edge $e = \{u, v\} \in E$, we create two edges in $E^*$: one edge $e' = \{u', v'\}$ and one edge $e'' = \{u'', v''\}$. We negate the weights for those edges, i.e. $w(e') = w(e'') = -w(e)$. Now $G^*$ contains two copies of $G$ with negated edge-weights. In a last step, for every node $v \in V$, we add an edge $e_v = \{v', v''\}$ to $E^*$ and set its weight to 0.

Show that a minimum perfect matching $M^*$ in $G^*$ induces a maximum weighted matching $M$ in $G$ and vice versa.

Problem 3: Blossom Algorithm

Apply Edmonds’ blossom algorithm to augment the given matchings in the graphs below. Use $v_1$ as starting node on your search for blossoms. Draw the network after each contraction-step.

(a)  
(b)