Assignment 6

Post Date: 30 May 2014   Due Date: 6 June 2014   Tutorial: 11 June 2014
You are permitted and encouraged to work in groups of two.

Problem 1: Dual to Max-Flow 5 Points

Let $P$ be the linear program of a Max-$s$-$t$-Flow problem formulated as a Min-Cost-Flow problem by adding an additional edge $(t, s)$ with costs $-1$ to a network (see the lecture notes).

(a) Build the dual problem $D$ of $P$.

(b) Show that an optimum solution of $D$ can be interpreted as a Min-$s$-$t$-Cut.

Problem 2: Soccer-League-Problem 7 Points

We are given a soccer league with $n$ teams. At one point in time during the season, the matrix $G$ indicates how many games between the teams are left to play, i.e. team $i$ still has $g_{ij}$ games to play against team $j$. The vector $p$ indicates the scores of the teams at the given point in time, i.e. team $i$ has $p_i$ points. Contrary to the actual point-scheme of the Bundesliga, winning a match gives 2 points, a tie 1 point and loosing a match 0 points.

Consider now the following problem: How can we tell if team $i$ has still a chance of winning the league (i.e. having the most points at the end)?

Reduce the problem to a maximum flow problem.

Problem 3: Goldberg-Tarjan 8 Points

Find the maximum flow in the network below using the algorithm of Goldberg and Tarjan ($s = v_1, t = v_6$).