Assignment 6

Post Date: 3 July 2013   Tutorial: 12 July 2013

Problem 1: Edge Separators 2 Points

Show that there is a constant $c$ and a natural number $n_0$ such that for any biconnected planar graph $G = (V, E)$ with $n \geq n_0$ vertices and maximum vertex degree $\Delta$ there is a set $S \subset V$ with $1/3n \leq |S| \leq 2/3n$ such that $|E(S, V \setminus S)| \leq c\sqrt{\Delta n}$.

Problem 2: Girth 6 Points

The girth of a graph is the number of edges on a shortest cycle.

(a) For a given vertex $v$, show how to compute a shortest cycle that contains $v$.

(b) Use (a) to compute the girth of a graph. What is the run time of your algorithm?

(c) Use the planar separator theorem to improve the run time of your algorithm if the input is restricted to planar graphs.

Problem 3: Separators of Trees 4 Points

Let $T$ be a tree with non-negative weights on the vertices that sum to one. A weighted vertex separator of $T$ is a partition of the vertex set into two sets $A$ and $B$ of weight at most $2/3$ and a vertex $v$ such that there is no edge between $A$ and $B$.

(a) Show how to compute a weighted vertex separator of a tree in linear time.

(b) Can the vertex set of any tree with non-negative weights on the vertices summing to one be partitioned into two sets $A$ and $B$ of weight at most $1/2$ and a vertex $v$ such that there is no edge between $A$ and $B$?

Problem 4: Dual Complement of a Tree 2 Points

Let $T$ be a spanning tree of a connected graph $G$, let $G^*$ be the geometric dual of $G$ and let $T^*$ be the subgraph of $G^*$ induced by the dual edges of the edges not in $T$. Prove that $T^*$ is a tree.

[please turn over]
Problem 5: Separators in Grids 6 Points

Let $G$ be an $n \times n$ grid. Grow a down-first BFS-tree $T$ from the upper left corner of $G$.

(a) What is $\text{diam}(T)$?
(b) What is $\Delta^T_G$?
(c) Assuming $n = 5$ and all faces having weight $1/17$ demonstrate how according to the proof of Lemma 4.3 a weighted cycle separator with at most $\text{diam}(T) + \Delta^T_G$ edges is found.