Assignment 4

Post Date: 5 June 2013  Tutorial: 14 June 2013

Problem 1: Selfdual

A plane graph $G$ is selfdual if $G$ is isomorph to its geometric dual $G^*$.

(a) Show that a selfdual graph with $n$ vertices has $2n - 2$ edges.

(b) Construct for each $n \geq 4$ a selfdual graph $G_n$.

Problem 2: Constructing the Dual

Show how to construct in linear time the geometric dual of a plane graph.

Problem 3: Combinatorial Dual

A multi-graph $G^*$ is a combinatorial dual of a multi-graph $G$, if there is a one-to-one correspondence between the edges of $G$ and the edges of $G^*$ such that the simple cycles of $G$ correspond to the minimal cuts of $G^*$. When contracting an edge $e$ of a multi-graph $G$, we merge the end vertices $v$ and $w$ of $e$ into a new vertex $u$, we delete $e$, but we keep all other edges that were incident to $v$ or $w$. See the figure below.

Let $e$ be an edge of a multi-graph $G$ that is not a loop. Let $G^*$ be a combinatorial dual of $G$ without isolated vertices. Show that

(a) $G^*$ does not have to be connected even if $G$ is.

(b) $G^*$ is biconnected if $G$ is.

(c) $G^* - e^*$ is a combinatorial dual of $G/e$.

Problem 4: Cuts

Let $C_S = E(S, V \setminus S)$ and $C_T = E(T, V \setminus T)$ be two cuts of a graph. Show that

(a) $C_S \cup C_T$ is a cut if $C_S \cap C_T = \emptyset$.

(b) $C_S \setminus C_T$ is a cut if $C_T \subseteq C_S$.

(c) a set of edges is a cut if and only if it is the union of some edge disjoint minimal cuts.