Assignment 7

Available Since: June 01, 2011  Due Date: June 08, 2011, 2:30 p.m.

You are permitted and encouraged to work in groups of two.

Exercise 1: Euler’s Formula 6 Points

Show that
\[ n - m + f = 1 + k \]
for a planar simple undirected graph with \( n \) vertices, \( m \) edges, \( f \) faces, and \( k \) connected components and conclude that
\[ m \leq 3n - 6 \]
if \( n \geq 3 \).

Is there a planar simple undirected graph in which every vertex has degree higher than five?

Exercise 2: Canonical Ordering 6 Points

Let \( G = (V, E) \) be a planar triangulated graph with \( n \geq 3 \) vertices. An ordering \( v_1, \ldots, v_n \) of the vertices of \( G \) is a canonical ordering if \( v_1, v_2, v_n \) are the vertices on the outer face of \( G \) and for each \( j = 3, \ldots, n - 1 \), there are
\[ 1 \leq i_1 < i_2 < j < k \leq n \] such that \( \{v_{i_1}, v_j\}, \{v_{i_2}, v_j\}, \{v_j, v_k\} \in E \).

Show that a triangulated planar graph always admits a canonical ordering.

**Hint:** Consider inductively for \( i = n - 1, \ldots, 3 \) the set \( C_i \) of vertices on the outer face of the graph induced by \( V \setminus \{v_{i+1}, \ldots, v_n\} \). Choose \( v_i \in C_i \setminus \{v_1, v_2\} \) such that \( v_i \) is adjacent to exactly two vertices of \( C_i \).